

24002

B. Tech. 1st Semester Examination,

December-2012

MATHEMATICS-I

Paper-Math.-101-F

Time allowed : 3 hours]

[Maximum marks : 100

Note : Attempt five questions in total, selecting one question from each section. Question No. 1 is compulsory.

1. (a) Test the convergence of the series

$$\sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{1}{n}$$

(b) Give the statement of D' Alembert's ratio test and Gauss Test.

(c) Define elementary matrices with example.

(d) Find the eigen value of the matrix

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

24002-P-7-Q-9 (12)-30650

24002

[P.T.O.]

- (e) Expand  $\tan x$  in powers of

$$\left(x - \frac{\pi}{4}\right) \text{ by using Taylor's series.}$$

- (f) If  $u = e^{xyz}$ , then find

$$\frac{\partial^3 u}{\partial x \partial y \partial z}$$

- (g) Change the order of integration in

$$\int_0^a \int_y^a \frac{x \, dx \, dy}{x^2 + y^2}.$$

- (h) Evaluate :

$$\iint xy (x^2 + y^2)^{3/2} \, dx \, dy \text{ over the positive quadrant of the circle } x^2 + y^2 = 1. \quad 20$$

### Section-A

2. (a) Test the convergence of the series

$$\sum (\sqrt{n^4 + 1} - \sqrt{n^4 - 1}) \quad 6$$

- (b) Test the convergence of the series

$$\sum_{n=1}^{\infty} \frac{x^{n+1}}{(n+1)\sqrt{n}} \quad 6$$

- (c) Test the convergence of the series

$$\sum \frac{(n+1)^n x^n}{n^{n+1}} \quad 8$$

3. (a) Define absolute convergence of a series and hence test the convergence and absolute convergence of the series :

$$\frac{1}{2 (\log 2)^p} - \frac{1}{3 (\log 3)^p} + \frac{1}{4 (\log p)^p} - \dots \infty (p > 0).$$

12

- (b) Discuss the convergence of the series :

$$\frac{x}{1} + \frac{1}{2} \frac{x^3}{3} + \frac{1 \cdot 3}{2 \cdot 4} \cdot \frac{x^5}{5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \cdot \frac{x^7}{7} + \dots (x > 0). \quad 8$$

### Section-B

4. (a) For the matrix A find non-singular matrices P and Q such that PAQ is in the normal form. Hence find the rank of A. Where matrix

$$A = \begin{bmatrix} 2 & 1 & -3 & 6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix} \quad 10$$

- (b) Using the matrix method, show that the equations

$$3x + 3y + 2z = 1,$$

$$x + 2y = 4,$$

$$10y + 3z = -2,$$

$$2x - 3y - z = 5$$

are consistent and hence obtain the solutions for

$x, y$  and  $z$ .

10

5. (a) Verify Cayley-Hamilton theorem for the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}. \text{ Hence compute } A^{-1}$$

8

- (b) Diagonalise the matrix

$$A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

and hence find  $A^4$ .

12

## Section-C

6. (a) If  $y = e^{m \cos^{-1} x}$ , Prove that

$$(1-x^2) y_{n+2} - (2n+1) x y_{n+1} - (n^2 + m^2) y_n = 0 \quad 7$$

- (b) Calculate the approximate value of  $\sqrt{10}$  to four decimal places by taking first four terms of an appropriate Taylor's series. 7

- (c) Find all the asymptotes of the curve

$$x^3 + 2x^2y - xy^2 - 2y^3 + 3xy + 3y^2 + x + 1 = 0 \quad 6$$

7. (a) If

$$u = \sin^{-1} \frac{x+y}{\sqrt{x} + \sqrt{y}}$$

Prove that :

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = - \frac{\sin u \cos 2u}{4 \cos^3 u}$$

(b) Evaluate :

$$\int_0^{\infty} \frac{\tan^{-1} ax}{x(1+x^2)} dx \quad (a \geq 0)$$

by applying differentiation under the integral sign.

10

### Section-D

8. (a) Express :

$\int_0^1 x^m (1-x^n)^k dx$  in terms of gamma function and hence evaluate

$$\int_0^1 x^5 (1-x^3)^{10} dx.$$

10

(b) Find the volume of the solid obtained by revolving  $x = a \cos \theta$ ,  $y = b \sin \theta$  about the y-axis.

10

9. (a) Evaluate :

$\iint r^3 dr d\theta$ , over the area bounded between the circle  $r = 2 \cos \theta$  and  $r = 4 \cos \theta$

10

(b) Evaluate :

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dz \, dy \, dx}{\sqrt{1-x^2-y^2-z^2}},$$

by changing to spherical polar co-ordinates.

10